M.Sc. Sem I
Paper I

Group Action

A group action is a representation of the elements of a group as symmetries of a set. Many groups have a natural group action coming from their construction; e.g. the dihedral group D_4 acts on the vertices of a square because the group is given as a set of symmetries of the ~~space~~. Square.

A group action of a group on a set is an abstract generalization of this idea, which can be used to derive useful facts about both the group and the set it acts on.

Formally, a group action of a group G on a set X is a function $f : G \times X \to X$ satisfying the following properties.

i.) $f(e_G, x) = x \quad \forall x \in X$

ii.) $f(gh, x) = f(g, f(h,x)) \quad \forall g, h \in G, \; x \in X$

When the action is clear, the function $f(g,x)$ is often written as $g \cdot x$. With this ~~contradiction~~ notation the axioms become

1.) $e_G \cdot x = x$.

2.) $g \cdot (h \cdot x) = (gh) \cdot x$.

Example. The standard example of a group action is when G equals the symmetric group S_n (or a subgroup of S_n) and $X = \{1, 2, \dots, n\}$. Then G acts on X by the formula $g \cdot x = g(x)$. The properties are clear $e \cdot x = e(x) = x$ where e is the identity of S_n and $g \cdot (h \cdot x) = g \cdot h(x) = g\,h(x) = g(h(x)) = (g \circ h)(x)$.